Notes

Shear Layer Flow Induced by Blunt-Nosed Bodies

L. Filler*
Seattle University, Seattle, Wash.
AND
W. Simon†
Martin Marietta Corporation, Denver, Colo.

Nomenclature

C_{DN} = nose drag coefficient
C_p = specific heat at constant pressure
M = Mach number
n = coordinate normal to streamline
p = pressure
r_B = body radius
R_N = nose radius
R_S = shock wave radius of curvature
S = entropy
t = body thickness
T = temperature
u = velocity
γ = ratio of specific heats
$\delta(\epsilon)$ = shock wave stand-off distance
ϵ = density ratio across normal shock wave
ζ = pressure ratio across normal shock wave
ρ = density

Subscripts

0 = total conditions
∞ = freestream conditions
B = body surface
NS = downstream of normal shock wave

It is well known that the curved shock wave supported by a blunt-nosed body introduces vorticity in the flow field between the body and the shock wave. This inviscid, nonuniform flow field called the shear layer persists far downstream from the nose region of the body and affects both the heat transfer to the body and boundary-layer transition.[1] Numerous authors[2–4] have investigated boundary-layer flow in the presence of an external shear flow of constant vorticity. Another equally important consequence of the shear flow is its effect on the stability of flared blunt-nosed bodies and of the effectiveness of aerodynamic controls.[5]

The present analysis is based on an assumed form of the entropy distribution in the shear layer which is independent of the amount of nose bluntness. The insensitivity of the shock wave shape to changes in nose bluntness permits this degree of generality. For a given freestream Mach number, the assumed entropy distribution must satisfy at least two conditions: the vorticity at the body surface must be given correctly, and the integrated entropy distribution must give the correct value of nose drag coefficient for a prescribed nose shape.

The velocity distribution in a constant pressure shear layer is given as a function of the entropy distribution by

$$u(n) = \left\{2C_p\left[T_0 - T_\infty \exp\left(\frac{S(n) - S_\infty}{C_p}\right)\right]\right\}^{1/2} \tag{1}$$

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* Associate Professor, Mechanical Engineering. Member AIAA.

† Associate Research Scientist, Aeronautics Department. Member AIAA.

The choice of entropy distribution in the shear layer is based on the mode by which the bow shock wave introduces vorticity in the flow field. The streamlines that enter the shock wave near the nose of the body will lie close to the body surface downstream of the nose. The bow shock wave is curved highly in the vicinity of the blunt nose, and the largest amount of vorticity is introduced at the nose region. Hence, far downstream from the nose the streamlines adjacent to the body surface will carry the bulk of the vorticity introduced by the bow shock wave. Away from the nose region the shock wave is less curved, and the streamlines leaving the shock at this location carry less vorticity. An entropy distribution is assumed which exhibits this characteristic behavior, namely,

$$S(n) = S_\infty + (S_{NS} - S_\infty) \times \exp-\left[\omega_1\left(\frac{n - r_B}{r_B}\right) + \omega_2\left(\frac{n - r_B}{r_B}\right)^2\right] \tag{2}$$

Here, S_{NS} is the entropy carried by the stagnation streamline, r_B is the body radius, and ω_1 and ω_2 are coefficients to be determined. The assumed form of the entropy distribution decays exponentially from $S(n) = S_{NS}$ at $n = r_B$ to $S(n) = S_\infty$ as $n \rightarrow \infty$. The coefficients ω_1 and ω_2 are to be determined such that the vorticity at the body surface is given correctly and the entropy distribution gives the correct value for the nose drag coefficient.

The velocity distribution made dimensionless with respect to freestream quantities now may be written as

$$\frac{u(n)}{u_\infty} = \left\{\frac{(T_0/T_\infty) - \zeta_0^{[(\gamma-1)/\gamma]\exp-f(n)}}{(T_0/T_\infty) - 1}\right\}^{1/2} \tag{3}$$

where

$$f(n) = \omega_1\left(\frac{n - r_B}{r_B}\right) + \omega_2\left(\frac{n - r_B}{r_B}\right)^2$$

is a dimensionless scale variable and $\zeta_0 = p_{0,\infty}/p_{0,NS}$ is the total pressure jump across the normal shock wave on the stagnation streamline. The temperature and density distributions are given by

$$T(n)/T_\infty = \rho_\infty/\rho(n) = \zeta_0^{[(\gamma - 1)/\gamma]\exp-f(n)} \tag{4}$$

From the dimensionless scale variable $f(n)$, one finds

$$\frac{n}{r_B} = 1 - \frac{\omega_1}{2\omega_2} + \left[\left(\frac{\omega_1}{2\omega_2}\right)^2 + \frac{f(n)}{\omega_2}\right]^{1/2} \tag{5}$$

Thus, the shear layer properties can be found as functions of $f(n)$, whereas the distance into the flow field at which these properties are obtained is given by Eq. (5), once ω_1 and ω_2 have been determined.

On differentiating Eq. (3) and evaluating the derivative at $f(n) = 0$, the first coefficient can be shown to be

$$\omega_1 = \frac{r_B u_B(\partial u/\partial n)_B}{C_pT_\infty\zeta_0^{[(\gamma-1)/\gamma]}\ln\zeta_0^{[(\gamma-1)/\gamma]}} \tag{6}$$

The second coefficient is obtained by evaluating the second derivative of Eq. (3) at $f(n) = 0$:

$$\omega_2 = \frac{r_B^2}{2}\left\{\frac{u_B(\partial^2 u/\partial n^2)_B}{C_pT_\infty\zeta_0^{[(\gamma-1)/\gamma]}\ln\zeta_0^{[(\gamma-1)/\gamma]}} + \left(\frac{\omega_1}{r_B}\right)^2 \times \left[\frac{C_pT_\infty\zeta_0^{[(\gamma-1)/\gamma]}\ln\zeta_0^{[(\gamma-1)/\gamma]}}{u_B^2} + (\ln\zeta_0^{[(\gamma-1)/\gamma]} + 1)\right]\right\} \tag{7}$$

Consequently, ω_1 is given by the velocity and velocity gradient at the body surface. Although ω_2 is given by the velocity and its second derivative at the body surface, it is dependent also on ω_1. But the velocity gradient at the body surface is just the vorticity at the surface, hence, ω_1 is to be evaluated so that the assumed entropy distribution gives the correct value of vorticity at the body surface. In place of Eq. (7) the coefficient ω_2 is evaluated so that the assumed entropy distribution gives the correct nose drag. Then the second derivative of the velocity at the surface must be accepted from Eq. (7), with ω_2 determined from the nose drag coefficient.

In order to evaluate ω_1, the velocity and the velocity gradient at the body surface are required. The velocity u_B at the surface is found readily from Eq. (3), with $f(n) = 0$. It is simply the velocity obtained through an isentropic expansion from the stagnation conditions behind the normal shock wave to the freestream pressure in the flow field far downstream from the nose region. At the body surface the velocity gradient is found from the surface vorticity. For two-dimensional flow the vorticity at the surface is zero, hence, $\omega_1 = 0$. For three-dimensional flow the calculation of the surface vorticity introduces the shock wave radius of curvature on the symmetry line. It can be determined readily for spherical shock waves, and, to this approximation, the method is not accurate for general nose shapes that do not support spherical shock waves. However, the insensitivity of the shock wave shape to changes in nose bluntness permits a large degree of generality. Thus, for three-dimensional flow the velocity gradient at the surface is[6]

$$\left(\frac{\partial u}{\partial n}\right)_B = \frac{(1-\epsilon)^2}{\epsilon}\,\frac{p_B}{p_{NS}}\,\frac{r_B\,u_\infty}{R_S{}^2}$$

where $\epsilon = \rho_\infty/\rho_{NS}$, and R_S is the shock wave radius of curvature. The shock wave radius of curvature may be calculated from[7]

$$\frac{R_S}{R_N} = \frac{1}{\Gamma}\,\frac{\delta(\epsilon)}{R_N}\left[1 + \frac{\delta(\epsilon)}{R_N}\right]$$

where

$$\Gamma = \frac{2(\gamma+1) + 2\gamma(\gamma+1)M_\infty{}^2 - (\gamma+1)M_\infty{}^2\{(\gamma+1)^2M_\infty{}^2/[4\gamma M_\infty{}^2 - 2(\gamma-1)]\}^{1/(\gamma-1)}}{4\gamma(M_\infty{}^2-1)}$$

and $\delta(\epsilon)/R_N$ is the ratio of the shock wave stand-off distance to the nose radius which may be estimated best by[8]

$$\delta(\epsilon)/R_N = \epsilon[1 - (\tfrac{8}{3}\epsilon)^{1/2} + 3.6\epsilon]$$

Thus, with $p_B = p_\infty$ one finds

$$u_B\left(\frac{\partial u}{\partial n}\right)_B = \frac{A(r_B/R_N)u_B{}^2}{(u_B/u_\infty)R_N}$$

where

$$A = \frac{(1-\epsilon)^2}{\epsilon}\,\frac{p_\infty}{p_{NS}}\left\{\frac{1}{\Gamma}\,\frac{\delta(\epsilon)}{R_N}\left[1 + \frac{\delta(\epsilon)}{R_N}\right]\right\}^2$$

Finally, ω_1 is found from Eq. (6) to be

$$\omega_1 = \frac{2A(r_B/R_N)^2}{u_B/u_\infty}\left[\frac{T_0 - T_\infty\zeta_0{}^{[(\gamma-1)/\gamma]}}{T_\infty\zeta_0{}^{[(\gamma-1)/\gamma]}\ln\zeta_0{}^{[(\gamma-1)/\gamma]}}\right] \qquad (8)$$

Under the assumption that the flow field streamlines are straight lines everywhere parallel to the afterbody surface, the nose drag coefficient based on body thickness t in the two-dimensional and on afterbody cross-section area in the three-dimensional case is given by

$$\frac{C_{D_N}}{4} = \int_{1/2}^{\infty}\frac{\rho u}{\rho_\infty u_\infty}\left(1 - \frac{u}{u_\infty}\right)d\left(\frac{n}{t}\right) \qquad \text{(two-dimensional)} \qquad (9a)$$

$$\frac{C_{D_N}}{4} = \int_{1}^{\infty}\frac{\rho u}{\rho_\infty u_\infty}\left(1 - \frac{u}{u_\infty}\right)\frac{n}{r_B}\,d\left(\frac{n}{r_B}\right) \qquad \text{(three-dimensional)} \qquad (9b)$$

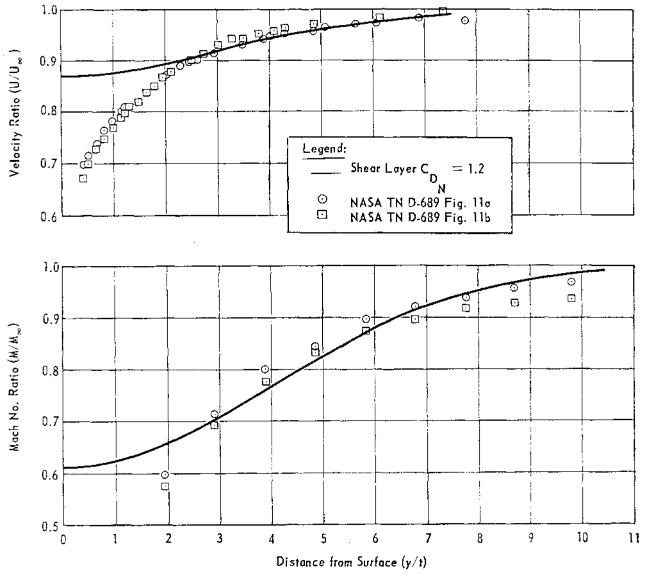

Fig. 1 Shear layer velocity and Mach number profiles on blunted flat plate at $M = 4.7$.

For two-dimensional flow, Eq. (5) and the condition $\omega_1 = 0$ yield

$$\frac{n - (t/2)}{t} = \left[\frac{f(n)}{\omega_2}\right]^{1/2} \qquad (10)$$

Let $Q(f) = \rho u/\rho_\infty u_\infty[1 - (u/u_\infty)]$ and transform the variable of integration in Eq. (9a) to f. There then results, upon integration by parts

$$\omega_2 = 2(I/C_{D_N})^{1/2} \qquad (11)$$

where

$$I = \int_0^{Q_B} f^{1/2}dQ$$

For a given freestream Mach number and ratio of specific heats, $Q(f)$ can be calculated from Eqs. (3) and (4). The integral I can be calculated numerically, and for a prescribed nose drag coefficient ω_2 is found from Eq. (11). The distance from the body surface at which the shear layer flow field properties are obtained for a fixed $f(n)$, then, is given by Eq. (10).

In the three-dimensional case, a change of integration variable as for the two-dimensional case gives

$$\frac{C_{D_N}}{4} = \frac{1}{2\omega_2}\int_0^\infty Q\,df + \frac{1}{2\omega_2}\left(\frac{2\omega_2}{\omega_1} - 1\right)\times \int_0^\infty \frac{Q\,df}{\{1 + [4\omega_2/(\omega_1)^2]f(n)\}^{1/2}}$$

which gives

$$\omega_2 = \frac{1}{2}\,\frac{I_1 - I_2(\omega_1,\omega_2)}{(C_{D_N}/4) - [I_2(\omega_1,\omega_2)/\omega_1]} \qquad (12)$$

where

$$I_1 = \int_0^\infty Q\,df \qquad I_2 = \int_0^\infty \frac{Q\,df}{\{1 + [4\omega_2/(\omega_1)^2]f(n)\}^{1/2}}$$

When $Q(f)$ is calculated at specified values of f from Eqs. (3) and (4), ω_2 can be found from Eq. (12) for a prescribed nose

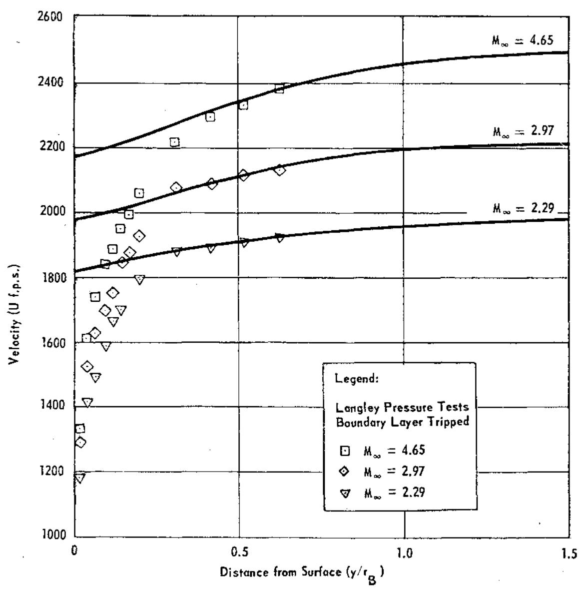

Fig. 2 Shear layer velocity profiles on blunted cylinder $r_B/R_N = 1.667$, $C_{DN} = 0.25$.

drag coefficient. Because ω_2 appears in the integrand of I_2, an iterative process is required for the calculation. ω_1 is found from Eq. (8) for prescribed values of M_∞, γ, and r_B/R_N. The calculated values of ω_1 and ω_2 together with Eq. (5), then, give the distance from the body surface at which the shear layer properties are obtained.

Experimental data suitable for checking the present method generally are not available, particularly at the higher Mach numbers. For the two-dimensional case Ref. 9 gives velocity and Mach number profiles at a Mach number of 4.7. The data of Ref. 9 for the 0.062-in. leading edge radius are plotted in Fig. 1 for two downstream stations, 0.933 and 1.099 ft. Velocity and Mach number distributions given by the present method with $C_{DN} = 1.2$ for the cylindrically blunted nose also are shown and compare favorably with the experimental values outside the boundary layer. For the three-dimensional case, velocity profiles obtained at Mach numbers of 2.29, 2.97, and 4.65 shown in Fig. 2 are compared with experimental data obtained from wind-tunnel tests of a non-hemispherically blunted cylinder. The theoretical velocity distributions were obtained for $r_B/R_N = 1.667$ and a nose drag coefficient of 0.25, which is an average value for the range of Mach numbers presented. Again, the experimental points compare well with the theoretical curve outside the boundary layer.

Figure 3 shows typical shear layer Mach number profiles calculated at $M = 20$ for a hemispherically blunted cylinder at constant values of $\gamma = 1.2$, 1.4 ,and 1.67. Since the dynamic pressure is given by the square of the Mach number ratio, it is seen that, beyond about 1.5 body radii, the greatest defect in dynamic pressure occurs for frozen ($\gamma = 1.67$) flow. As far as 8 body radii from the surface, the dynamic pressure is only 39% of freestream for frozen flow but is 93% of freestream for equilibrium ($\gamma = 1.2$) flow. These results are indicative and must be qualified by the actual variation in γ as one moves outward in the flow from the body streamline. Nevertheless, the thermodynamic state of the gas incident upon control surfaces is demonstrably significant.

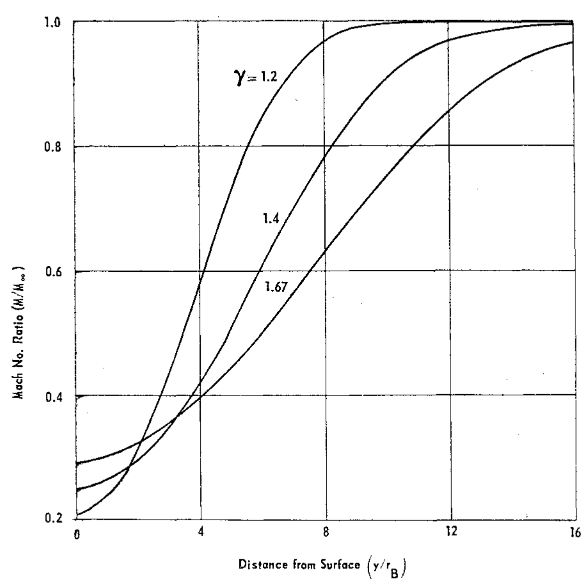

Fig. 3 Shear layer Mach number distribution at $M = 20$ on hemispherically blunted cylinder $C_{DN} = 0.9$.

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Manual Extraterrestrial Guidance and Navigational System

PAUL B. LIEBELT*

The Boeing Company, Seattle, Wash.

IT often is postulated that a fully or semiautomatic guidance and navigational system is required for lunar and solar system missions. If, however, a completely manual system could be devised, it could serve as a back-up system in case of automatic equipment failure, a monitor system for automatic equipment check, or a system to provide initial conditions for automatic equipment after a system shutdown for power conservation or repair.

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* Research Engineer, Electronics Technology Department, Aero-Space Division.